University of Mannheim
School of Social Sciences
Math Refresher for Political Science, Fall 2025
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Problem Set: Set Theory II

1. Let $S = \{\text{Homer, Marge, Bart, Lisa, Maggie}\}$. Enumerate the following relations.
   (a) "is a sibling of"
   (b) "is married to"
   (c) "is taller than"
   (d) "is older than"
2. Let $S = \{1, 2, 3, 4\}$. Graph the following relations.
   (a) $=$
   (b) $<$
   (c) $\{(1, 1), (2, 1), (2, 2), (3, 3), (4, 3), (4, 4)\}$
3. Determine the following sets.
   (a) The upper contour set of Lisa in 1c.
   (b) The lower contour set of Bart in 1d
   (c) The upper contour set of 2 in 2b
   (d) Let $S = \mathbb{R}$. Determine the upper contour set of $x \in S$ given the relation $R = \{(x, y) \in S \times S | x^2 - 1 = y\}$. Graph the binary relation. Is the upper contour set convex or concave or none of both?
4. Check whether the following relations are reflexive, irreflexive, transitive, complete, symmetric and asymmetric. Also check whether they are a weak, strict, weak partial or strict partial order (or none of those).
   (a) $\leq$
   (b) $<$
   (c) $=$
   (d) $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$
   (e) "was born before"
   (f) $\{(a, a), (a, b)\}$

5. Let $S = \{1, 2, 3\}$. Show by example that...

   (a) ...if R is asymmetric, it is also antisymmetric.

   (b) ...if R is asymmetric, it is also irreflexive.

   (c) ...if R is irreflexive and transitive, it is also asymmetric.

   (d) ...if R is symmetric and antisymmetric, it is also transitive.

6. Let $A = \{\{a\}, \{b\}, \{a, b\}\}$. Let $B = \mathcal{P}(A) \setminus \emptyset$, where $\mathcal{P}(A)$ is the *power set*, the set of all subsets of A. Define a binary relation $R \equiv \subseteq$.

   (a) Explicitly enumerate B, and state its cardinality.

   (b) Prove whether R is a weak order on B.

   (c) Prove whether R is a partial order on B.

   (a) Let R_1 and R_2 be transitive relations on a set S. Does it follow that $R_1 \cup R_2$ is transitive?

   (b) Let R_1 and R_2 be transitive relations on a set S. Does it follow that $R_1 \cap R_2$ is transitive?